Algebra 2 Solving Fractional Equations

To change an equation containing fractions to one that does not contain fractions you must multiply every term on both sides of the equation by the common denominator. This will cancel all denominators and leave you with a non-fractional equation.

* You must always check for extraneous roots. ("extra" roots that are roots of the derived equation, but not a root of the original equation)

Examples: Solve for all values of x that satisfy the equation. Be sure to check for extraneous roots. (The first one has been worked out for you.)

1. $\left(\frac{3}{2x}+\frac{1}{x}=\frac{1}{2}\right)\frac{2x}{1}$

$$\frac{3}{\cancel{2x}}\cdot\frac{\cancel{2x}}{1}+\frac{1}{\cancel{x}}\cdot\frac{\cancel{2}x}{1}=\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}x}{1}$$

$3+2=x$

$5=x$

$check: \frac{3}{10}+\frac{1}{5}=\frac{1}{2}$

2. $\frac{1}{12}+\frac{1}{y}=\frac{1}{4}$

$$\frac{1}{\cancel{12}}\cdot\boxed{\cancel{12}y}+\frac{1}{\cancel{y}}\cdot\boxed{12\cancel{y}}=\frac{1}{\cancel{4}}\cdot\boxed{\overset{3}{\cancel{12}}y}$$

$$y+12=3y$$
$$-y \qquad\quad -y$$

$$\frac{12}{2}=\frac{2y}{2}$$

$6=y$

$\boxed{\{6\}}$

Check:

$\frac{1}{12}+\frac{1}{6}=\frac{1}{4}$

$\frac{1}{4}=\frac{1}{4}$ ✓

3. $\frac{2}{x}+\frac{1}{4}=\frac{7}{x}$

$$\frac{2}{\cancel{x}}\cdot\boxed{4\cancel{x}}+\frac{1}{\cancel{4}}\cdot\boxed{\cancel{4}x}=\frac{7}{\cancel{x}}\cdot\boxed{4\cancel{x}}$$

$$\cancel{8}+x=28$$
$$\cancel{-8} \qquad -8$$

$x=20$

$\boxed{\{20\}}$

Check:

$\frac{2}{20}+\frac{1}{4}=\frac{7}{20}$

$\frac{7}{20}=\frac{7}{20}$ ✓

4. $\frac{1}{x}+\frac{1}{3}=\frac{3}{2x}-1$

$$\frac{1}{\cancel{x}}\cdot\boxed{6\cancel{x}}+\frac{1}{\cancel{3}}\cdot\boxed{\overset{2}{\cancel{6}x}}=\frac{3}{\cancel{2x}}\cdot\boxed{\overset{3}{\cancel{6x}}}-1\cdot\boxed{6x}$$

$$6+2x=9-\cancel{6}x$$
$$+6x \qquad +\cancel{6}x$$

$$\cancel{6}+8x=9$$
$$-\cancel{6} \qquad -6$$

$$\frac{8x}{8}=\frac{3}{8}$$

$\boxed{\left\{\frac{3}{8}\right\}}$

$x=\frac{3}{8}$ or $x=.375$

Sometimes you need to factor the denominator first to determine what to multiply by in factored form. (Problem number 5 has been solved for you.)

5. $\left(\frac{x}{x+3}=\frac{3}{x}+\frac{1}{x^2+3x}\right)\frac{x(x+3)}{1}$

$x^2=3(x+3)+1$

$x^2=3x+10$

$x^2-3x-10=0$

$(x-5)(x+2)=0$

$x-5=0; x+2=0$

$x=5; x=-2$

$x=5$

6. $\frac{x}{x+2}=\frac{3}{x}+\frac{4}{x^2+2x}$

$\frac{x}{x+2}=\frac{3}{x}+\frac{4}{x(x+2)}$

$\frac{x}{\cancel{x+2}}\cdot\boxed{x\cancel{(x+2)}}=\frac{3}{\cancel{x}}\cdot\boxed{\cancel{x}(x+2)}+\frac{4}{\cancel{x}\cancel{(x+2)}}\cdot\boxed{\cancel{x}\cancel{(x+2)}}$

$x^2=3x+6+4$

$x^2=3x+10$

$x^2-3x-10=0$

$(x-5)(x+2)=0$

$x-5=0$ | $x+2=0$

$x=5$ | $x=-2$ reject

$\boxed{\{5\}}$

7. $\frac{6}{x-3}=\frac{8x^2}{x^2-9}-\frac{4x}{x+3}$

$\frac{6}{x-3}=\frac{8x^2}{(x+3)(x-3)}-\frac{4x}{x+3}$

$\frac{6}{\cancel{x-3}}\cdot\boxed{\cancel{(x-3)}(x+3)}=\frac{8x^2}{\cancel{(x+3)}\cancel{(x-3)}}\cdot\boxed{\cancel{(x-3)}\cancel{(x+3)}}-\frac{4x}{\cancel{x+3}}\cdot\boxed{(x-3)\cancel{(x+3)}}$

$6x+18=8x^2-4x^2+12x$

$6x+18=4x^2+12x$

$0=4x^2+6x-18$

$0=2(2x^2+3x-9)$

$0=2(2x-3)(x+3)$

$2\neq 0$ | $2x-3=0$ | $x+3=0$

$\frac{2x}{2}=\frac{3}{2}$ | $x=-3$

$x=\frac{3}{2}$ | reject

$\boxed{\left\{\frac{3}{2}\right\}}$