

$$(f \circ g)(4) \quad f(g(4))$$

1. If $f(x) = x^2 - 4$ and $g(x) = 2x + 3$, write expressions for the following:

a. $(f+g)(2)$ $f(2) = (2)^2 - 4 = 0$ $0 + 7 = \boxed{7}$
 ~~$(f+g)(x)$~~
 $f(2) + g(2)$ $g(2) = 2(2) + 3 = 7$

b. $(f-g)(-3)$ $f(-3) = (-3)^2 - 4 = 5$ $5 - -3 = \boxed{8}$
 ~~$(f-g)(x)$~~
 $f(-3) - g(-3)$ $g(-3) = 2(-3) + 3 = -3$

c. $(f \cdot g)(4)$ $f(4) = (4)^2 - 4 = 12$ $(12)(11) = \boxed{132}$
 ~~$(f \cdot g)(x)$~~
 $f(4) \cdot g(4)$ $g(4) = 2(4) + 3 = 11$

d. $(\frac{f}{g})(5)$ $f(5) = (5)^2 - 4 = 21$ $\frac{21}{13}$
 ~~$(\frac{f}{g})(x)$~~
 $\frac{f(5)}{g(5)}$ $g(5) = 2(5) + 3 = 13$

e. $(\frac{g}{f})(-2)$ $g(-2) = 2(-2) + 3 = -1$ $\frac{-1}{0} = \boxed{\text{Undefined}}$
 ~~$(\frac{g}{f})(x)$~~
 $\frac{g(-2)}{f(-2)}$ $f(-2) = (-2)^2 - 4 = 0$

2. Let $f(x) = x^2 - 3x + 2$ and $g(x) = \sqrt{x-1}$. Evaluate each of the following:

a. $(f \circ g)(3)$ $3 \rightarrow g \xrightarrow{\sqrt{2}} f$
 $g(3) = \sqrt{3-1} = \sqrt{2}$
 $f(\sqrt{2}) = (\sqrt{2})^2 - 3(\sqrt{2}) + 2 = 2 - 3\sqrt{2} + 2 = \boxed{4 - 3\sqrt{2}}$

b. $(g \circ f)(3)$ $3 \rightarrow f \xrightarrow{2} g$
 $f(3) = (3)^2 - 3(3) + 2 = 2$
 $g(2) = \sqrt{2-1} = \sqrt{1} = \boxed{\pm 1}$

~~$(g \circ f)(3)$~~

3. If $f(x) = x^2 + 5$ and $g(x) = 3x - 2$, write expressions for the following:

f. $(f+g)(x)$
 $f(x) + g(x)$
 $(x^2 + 5) + (3x - 2)$
 $x^2 + 3x + 3$

g. $(f-g)(x)$
 $f(x) - g(x)$
 $(x^2 + 5) - (3x - 2)$
 $x^2 + 5 - 3x + 2$
 $x^2 - 3x + 7$

h. $(f \cdot g)(x)$
 $f(x) \cdot g(x)$
 $(x^2 + 5)(3x - 2)$
 $3x^3 - 2x^2 + 15x - 10$

	$3x$	-2
x^2	$3x^3$	$-2x^2$
$+5$	$+15x$	-10

i. $\left(\frac{f}{g}\right)(x)$
 $\frac{f(x)}{g(x)}$
 $\frac{x^2 + 5}{3x - 2}, x \neq \frac{2}{3}$

$3x - 2 \neq 0$
 $\frac{3x}{3} \neq \frac{2}{3}$
 $x \neq \frac{2}{3}$

j. $\left(\frac{g}{f}\right)(x)$
 $\frac{g(x)}{f(x)}$
 $\frac{3x - 2}{x^2 + 5}$

k. $f(g(-1))$ $-1 \rightarrow g \xrightarrow{-5} f$
 $g(-1) = 3(-1) - 2$
 $= -5$
 $f(-5) = (-5)^2 + 5$
 $= 30$

4. On a certain remote island in the Indian Ocean lives a population of stink beetles. The stink beetles are eaten by the rose-breasted cuckoo birds that nest on the island. The cuckoos, in turn, are eaten by the bald rock cats that prowl the island. If the population of beetles is x , the population of cuckoo birds can be represented by $b(x) = \frac{x}{200}$. Further, if the bird population on the island is x , the cat population can be represented by $c(x) = 3\sqrt{x} - 2$.

- a. If 20,000 stink beetles live on the island, what is the cuckoo population?

$$b(20,000) = \frac{20,000}{200}$$

$$= \boxed{100 \text{ cuckoo birds}}$$

- b. If 20,000 stink beetles live on the island, what is the cat population?

$$c(100) = 3\sqrt{100} - 2$$

$$= 3(10) - 2$$

$$= \boxed{28 \text{ cats}}$$

- c. Write a composition of functions using $b(x)$ and $c(x)$ to give the population of bald rock cats as a function of the stink beetle population.

$$c(b(x)) = 3\sqrt{\frac{x}{200}} - 2$$

↓ or

$$(c \circ b)(x)$$