5.2 Solve Linear Systems Graphically

Key Terms and Concepts

To solve a system of linear equations **graphically**, simply graph the two equations as lines in the same coordinate plane. The point (if any) where the two lines **intersect** is the solution. This is because the point of intersection is the only point that satisfies *both* equations.

Example: The lines $y=-x+5$ and $y=2x-4$ intersect at (3,2).
Therefore, the solution for the system of equations is $x=3$ and $y=2$.

If the two lines are parallel, they never intersect, and so there is no solution. If the two lines are identical (coincide), then there are infinitely many solutions.

Using a calculator, you can solve a system of equations by graphing both equations and then using the intersect feature.

Example: To solve the system, $y=-x+5$ and $y=2x-4$, graphically,

Press [Y=] – [ALPHA] [X] + 5 [ENTER]
2 [ALPHA] [X] – 4 [ENTER]
[2nd] [CALC] intersect
First curve? [ENTER] Second curve? [ENTER] Guess? [ENTER]

Model Problem:

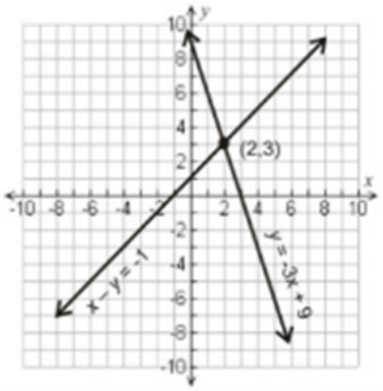
Solve the following system of equations graphically:

$x-y=-1$
$y=-3x+9$

Solution:

(A) $x-y=-1$
$-x \quad -x$
$\frac{-y}{-1}=\frac{-x-1}{-1}$
$y=x+1$

(B)



(C) Solution: (2, 3)

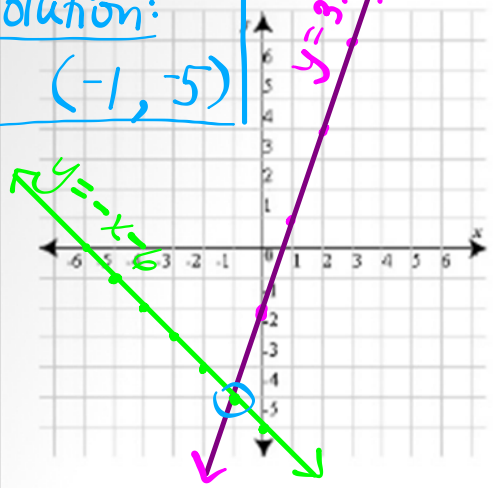
Explanation of steps:

(A) If either equation is not already in slope-intercept form, transform the equation by solving for y in terms of x.
(B) Graph both equations on the same coordinate plane, labeling each line.
(C) The point of intersection is the solution to the system of equations.
[A solution of (2,3) means $x=2, y=3$ solves both equations simultaneously.]

Practice Problems

1. Solve the system of equations graphically:

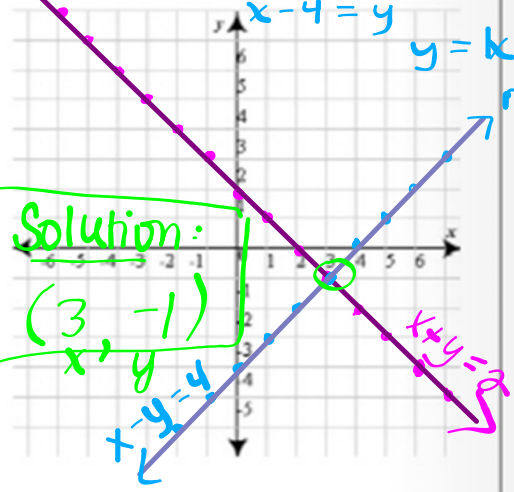
$y = 3x - 2$
$y = -x - 6$



Solution: $(-1, -5)$

2. Solve the system of equations graphically:

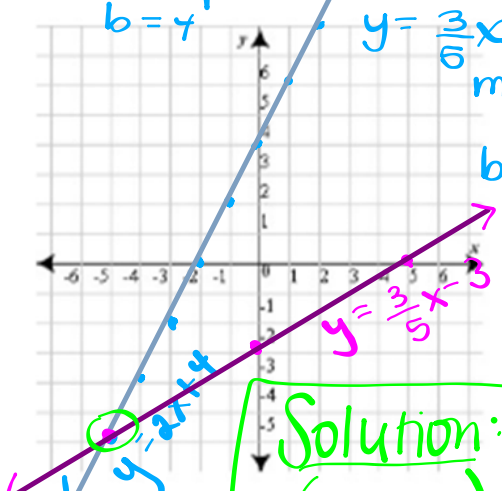
$x + y = 2$
$x - y = 4$



Solution: $(3, -1)$

3. Solve the system of equations graphically:

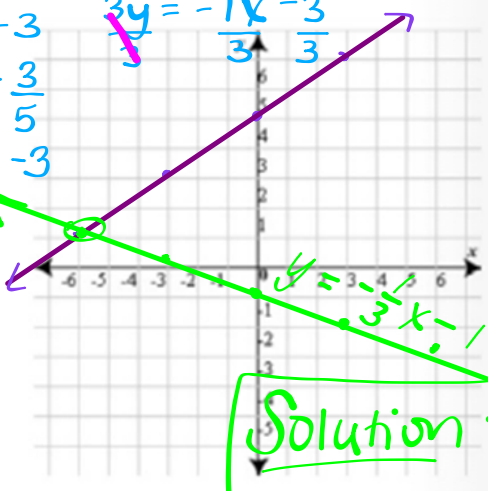
$3x - 5y = 15$
$y = 2x + 4$



Solution: $(-5, -6)$

4. Solve the system of equations graphically:

$y = \frac{2}{3}x + 5$
$x + 3y = -3$



Solution: $(-6, 1)$

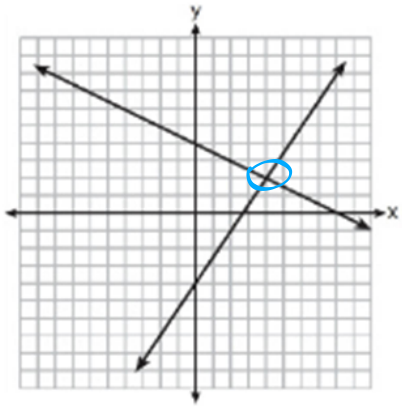
REGENTS QUESTIONS

Multiple Choice

1. If the lines whose equations are $x = -2$ and $y = 3$ were graphed on the same set of coordinate axes, their point of intersection would be
   (1) (–2,3)
   (2) (3,–2)
   (3) (2,–3)
   (4) (–3,2)

2. A system of equations is graphed on the set of axes below.

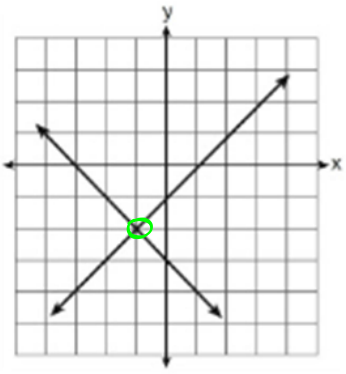
(x, y)
(4, 2)



The solution of this system is
   (1) (0,4)
   (2) (2,4)
   (3) (4,2)
   (4) (8,0)

3. What is the solution of the system of equations shown in the graph below?

(x, y)



   (1) (1,0) and (–3,0)
   (2) (0,–3) and (0,–1)
   (3) (–1,–2)
   (4) (–2,–1)

4. CC Two functions, $y=|x-3|$ and $3x+3y=27$, are graphed on the same set of axes. Which statement is true about the solution to the system of equations?

(1) (3,0) is the solution to the system because it satisfies the equation $y=|x-3|$.

(2) (9,0) is the solution to the system because it satisfies the equation $3x+3y=27$.

(3) (6,3) is the solution to the system because it satisfies both equations.

(4) (3,0), (9,0), and (6,3) are the solutions to the system of equations because they all satisfy at least one of the equations.

5. CC The line represented by the equation $4y+2x=33.6$ shares a solution point with the line represented by the table below.

x	y
−5	3.2
−2	3.8
2	4.6
4	5
11	6.4

The solution for this system is

(1) (−14.0, −1.4)
(2) (−6.8, 5.0)
(3) (1.9, 4.6)
(4) (6.0, 5.4)

6. CC What is the solution to the system of equations below?

$$y=2x+8$$
$$3(-2x+y)=12$$

(1) no solution
(2) infinite solutions
(3) (−1,6)
(4) $\left(\frac{1}{2},9\right)$

Constructed Response

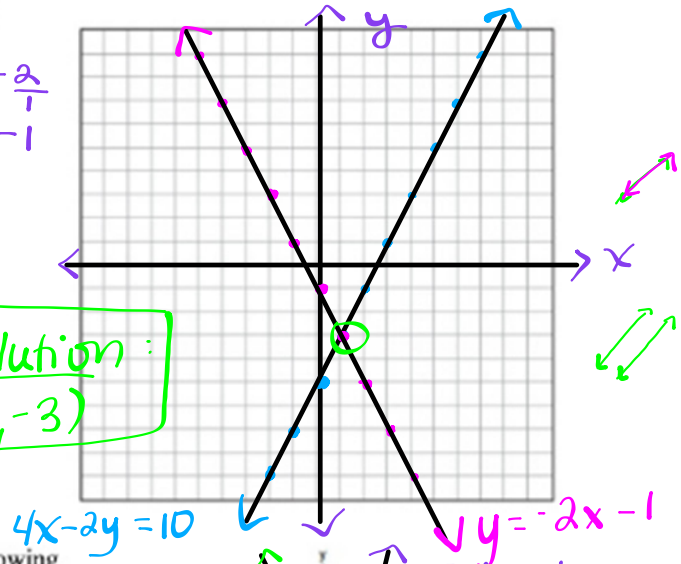
$y = mx + b$

7. On the grid below, solve the system of equations graphically for x and y.

$4x - 2y = 10$
$y = -2x - 1$

$m = -\frac{2}{1}$, $b = -1$

$4x - 2y = 10$
$-4x \quad -4x$
$-2y = -4x + 10$
$\frac{-2y}{-2} = \frac{-4x}{-2} + \frac{10}{-2}$
$y = 2x - 5$
$m = \frac{2}{1}$
$b = -5$



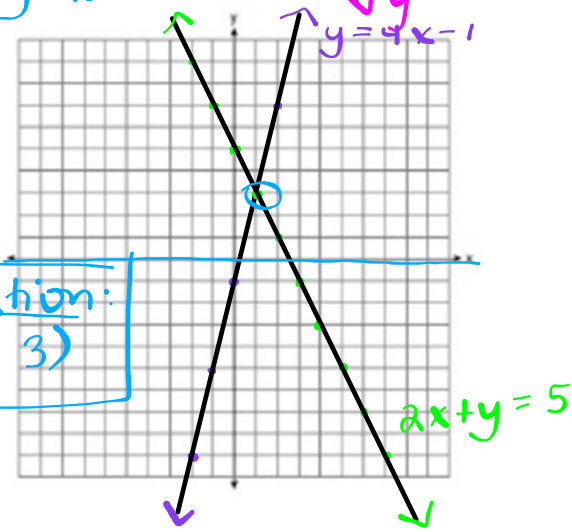
Solution: (1, -3)

8. On the set of axes below, solve the following system of equations graphically. State the coordinates of the solution.

$y = 4x - 1$
$2x + y = 5$

$m = \frac{4}{1}$, $b = -1$

$-2x \quad -2x$
$y = -2x + 5$
$m = -\frac{2}{1}$
$b = 5$



Solution: (1, 3)

9. (CC) In attempting to solve the system of equations $y = 3x - 2$ and $6x - 2y = 4$, John graphed the two equations on his graphing calculator. Because he saw only one line, John wrote that the answer to the system is the empty set. Is he correct? Explain your answer.

$y = mx + b$

$6x - 2y = 4$
$-6x \quad -6x$
$\frac{-2y}{-2} = \frac{-6x}{-2} + \frac{4}{-2}$
$y = 3x - 2$

$y = 3x - 2$

John is incorrect. These are the same line, there are infinitely many solutions.