

4.9 Graph Inequalities

Key Terms and Concepts

To graph a linear inequality, start by isolating (solving for) the variable y . Then, consider the graph of the equation that would result if the inequality symbol was replaced by an equal sign.

The points on this line are **included in the solution set** if the inequality symbol is \leq or \geq , but **not included in the solution set** if the inequality symbol is $<$ or $>$. To show inclusion (\leq or \geq), draw a **solid line**; otherwise, draw a **dashed line**.

\leq
 \geq
solid line

The line divides the plane into two parts. The part above the line includes all points where $y > mx + b$ and the part below the line includes all points where $y < mx + b$. So, if the inequality starts with $y >$ or $y \geq$, shade **above the line**; if it starts with $y <$ or $y \leq$, then shade **below the line**. Points in the shaded area are **included in the solution set**.

$<$
 $>$
dotted line

Special case: vertical lines

If the only variable in the inequality is x , solve for x and graph the corresponding vertical line (either solid or dashed according to the same rules). Then shade to the **right of the line** for $x >$ or $x \geq$, or to the **left of the line** for $x <$ or $x \leq$.

Example: The graph of $x \geq 5$ will have a solid vertical line at $x = 5$ and shading to the right of the line.



Inequalities can also be graphed on the calculator. Follow the same steps as entering an equation, but then change the symbol to the left of $Y_1 =$ from a line \backslash to a "shade above" ∇ or "shade below" \blacktriangledown symbol by moving to the line symbol and pressing **ENTER** repeatedly. You will still need to know whether to use a solid or dashed line when drawing your graph on paper, depending on the inequality symbol.



Model Problem:

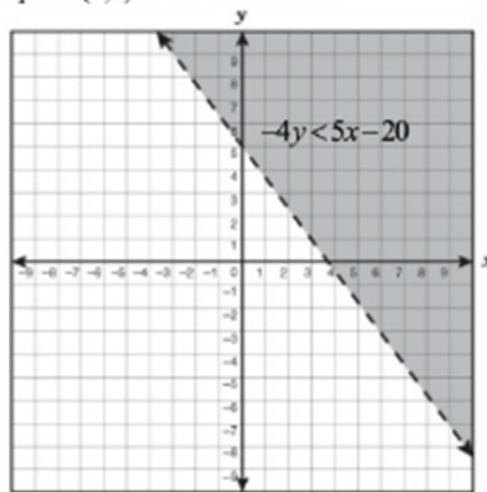
Graph the inequality $-4y < 5x - 20$. Is the point $(1,2)$ in the solution set?

Solution:

(A) Solving for y ,

$$\frac{-4y < 5x - 20}{-4 \quad -4}$$
$$y > -\frac{5}{4}x + 5$$

(B)



up, \geq
Shade up
 $<$, \leq
Shade down

(C) $(1,2)$ is not in the solution set.

Explanation of steps:

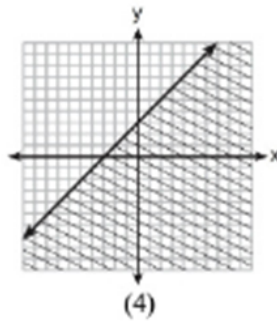
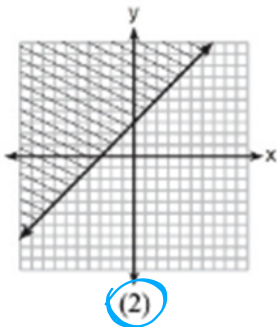
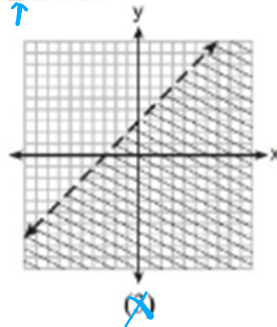
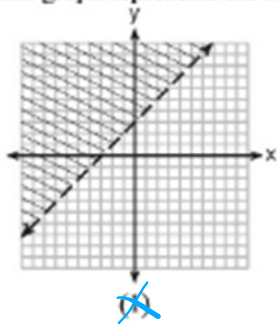
- (A) Solve the inequality for y . [Remember that multiplying or dividing both sides of an inequality by a negative value requires that you reverse the inequality symbol.]
- (B) Use the y -intercept and slope to graph the line. Use a solid line for \leq or \geq , or a dashed line for $<$ or $>$. Shade above the line if y is $>$ or \geq , or below the line if y is $<$ or \leq . [The $>$ means dashed and shaded above.]
- (C) If a point lies on a solid line or in a shaded area, it is in the solution set; otherwise, it is not. [$(1,2)$ lies in the unshaded area below the line, so it is not a solution.]

You could also graph $y > -\frac{5}{4}x + 5$ on the calculator:

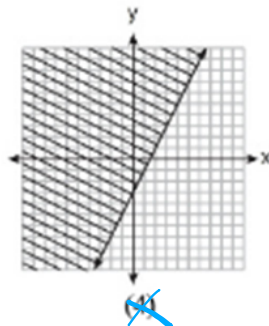
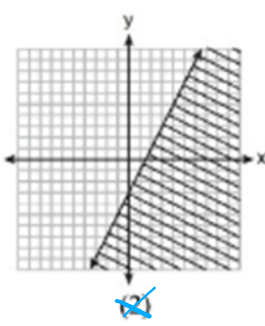
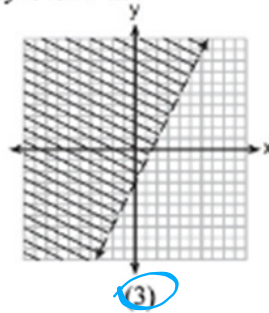
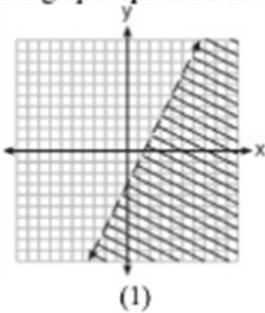


1. Press $\boxed{Y=}$ then the left arrow $\boxed{\leftarrow}$ twice until the cursor is over the $\boxed{\setminus}$ symbol.
2. Since the inequality starts with " $y >$ " press the $\boxed{\text{ENTER}}$ two times until the shade above $\boxed{\nabla}$ symbol appears. Move the cursor back to the right, after the $\boxed{=}$ sign.
3. Enter the right side of the inequality, $-5 \div 4$ $\boxed{\text{ALPHA}}$ $\boxed{[X]}$ $+ 5$ $\boxed{\text{ENTER}}$.
4. Press $\boxed{\text{ZOOM}}$ $\boxed{\text{ZStandard}}$.

3. Which graph represents the inequality $y \geq x + 3$?



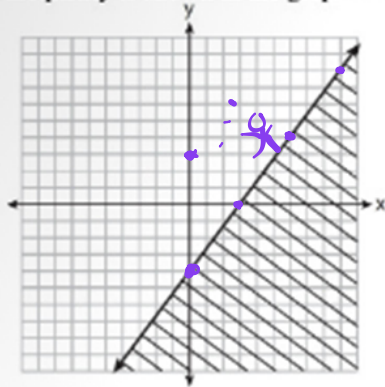
4. Which graph represents the solution of $2y + 6 > 4x$?



$$\begin{aligned} 2y + 6 &> 4x \\ \hline 2y &> 4x - 6 \\ \frac{2y}{2} &> \frac{4x - 6}{2} \\ y &> 2x - 3 \end{aligned}$$

(137)

5. Which inequality is shown in the graph below?



$$m = \frac{4}{3}$$
$$b = -4$$

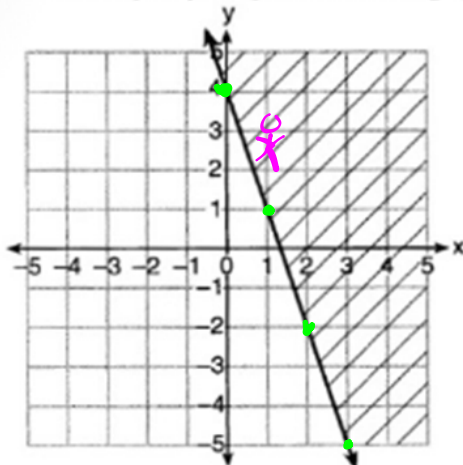
(1) $y \leq \frac{4}{3}x + 3$

(3) $y \leq \frac{4}{3}x - 4$

~~(2) $y \geq \frac{4}{3}x + 3$~~

~~(4) $y \geq \frac{4}{3}x - 4$~~

6. **CC** Which inequality is represented in the graph below?



$$m = -\frac{3}{1}$$
$$b = 4$$

(1) $y \geq -3x + 4$

(3) $y \geq -4x - 3$

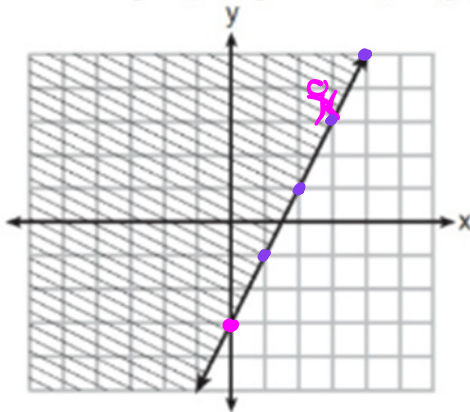
~~(2) $y \leq -3x + 4$~~

~~(4) $y \leq -4x - 3$~~

(138)

$y = mx + b$

7. (CC) Which inequality is represented by the graph below?



$m = \frac{2}{1}$

$b = -3$

- ~~(1) $y \leq 2x - 3$~~
- (2) $y \geq 2x - 3$

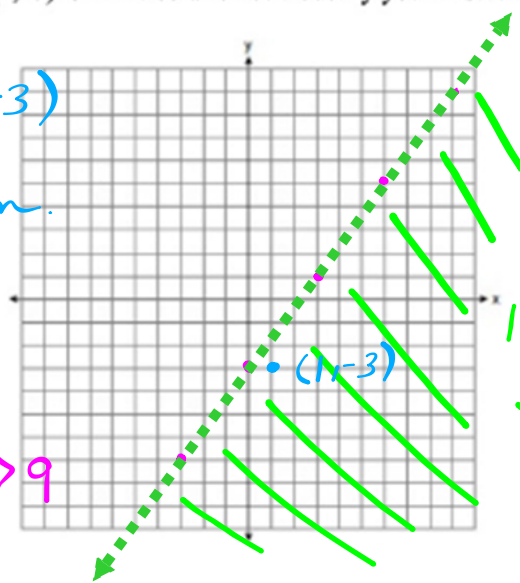
- ~~(3) $y \leq -3x + 2$~~
- ~~(4) $y \geq -3x + 2$~~

Constructed Response

8. Graph the solution set for the inequality $4x - 3y > 9$ on the set of axes below. Determine if the point $(1, -3)$ is in the solution set. Justify your answer.

$y = mx + b$

yes. $(1, -3)$ is a solution.



$4x - 3y > 9$

less than, shade down

$$\begin{array}{r} 4x - 3y > 9 \\ -4x \quad \quad -4x \\ \hline \end{array}$$

$$-3y > -4x + 9$$

$$\frac{-3y}{-3} > \frac{-4x + 9}{-3}$$

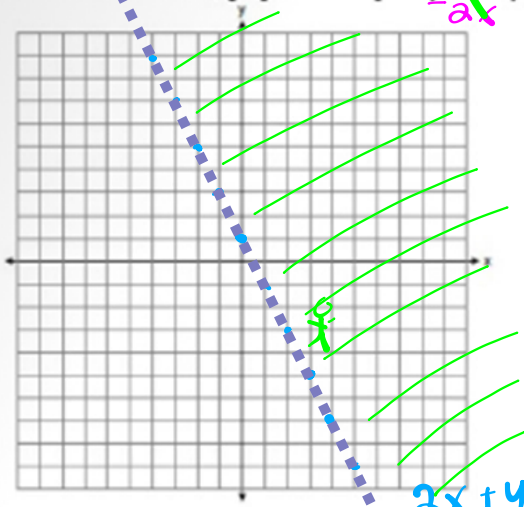
$$y < \frac{4}{3}x - 3$$

dotted line

$m = \frac{4}{3}$

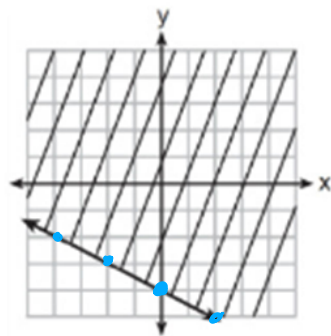
$b = -3$

9. **CC** On the set of axes below, graph the inequality $2x + y > 1$.



Shade up
dotted
 $y > -2x + 1$
 $m = -\frac{2}{1}$
 $b = 1$

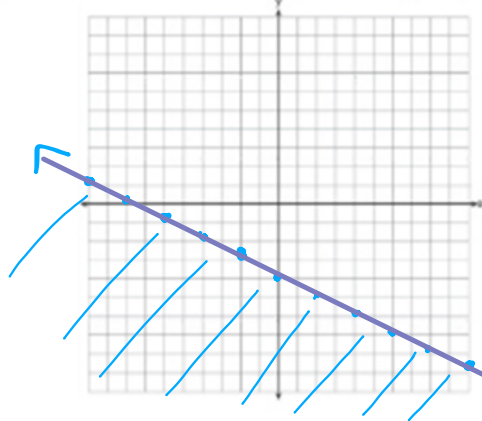
10. **CC** Shawn incorrectly graphed the inequality $-x - 2y \geq 8$ as shown below.



$-\frac{2}{2}y \geq \frac{1x + 8}{-2}$
 $y \leq -\frac{1}{2}x - 4$
solid
shade
 $m = -\frac{1}{2} \checkmark$
 $b = -4 \checkmark$

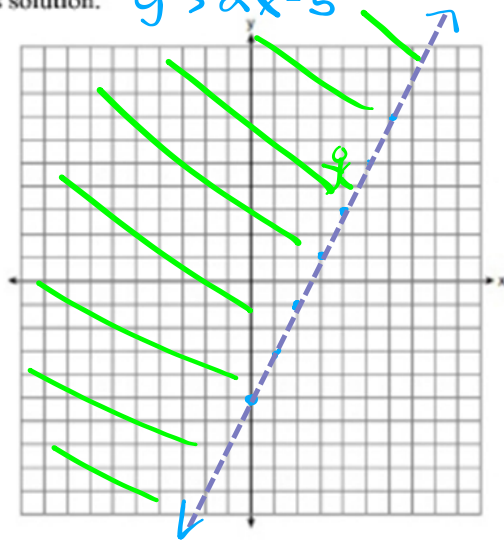
Shade down

Explain Shawn's mistake. Graph the inequality correctly on the set of axes below.



Shawn shaded incorrectly, he should have shaded down.

11. **CC** Graph the inequality $y > 2x - 5$ on the set of axes below. State the coordinates of a point in its solution. $y > 2x - 5$



$$m = \frac{2}{1}$$

$$b = -5$$

Solution:
 $(-4, -5)$

12. **CC** Graph the inequality $y + 4 < -2(x - 4)$ on the set of axes below.

$$y < -2x + 4$$



$$\frac{y + 4 < -2x + 8}{-4 \quad -4}$$

$$y < -2x + 4$$

dotted $m = \frac{-2}{1}$
 $b = 4$

