Exponential Growth and Decay Notes

Exponential Graphs Review:

**Growth:**
\[ y = a(1 + r)^x \]

**Decay:**
\[ y = a(1 - r)^x \]

*Any quantity that grows or decays by a fixed percent at regular intervals is said to possess exponential growth or exponential decay. When a quantity grows by a fixed percent at regular intervals, the pattern can be represented by the functions.*

\[ a = \text{initial amount before measuring growth/decay} \]
\[ r = \text{growth/decay rate (often a percent \( \rightarrow \) Change to a DECIMAL!)} \]
\[ x = \text{number of time intervals that have passed} \]

Example:
Malik bought a new car for $15,000. As he drove it off the lot, his best friend, Will, told him that the car's value just dropped by 15% and that it would continue to depreciate 15% of its current value each year. If the car's value is now $12,750 (according to Will), what will its value be after 5 years?

\[ y = a \left(1 - \frac{r}{100}\right)^t \]

\[ a = 15000 \]
\[ r = 15\% = .15 \]
\[ t = 5 \]

\[ y = \frac{12750 \times (1-.15)}{5} \]

\[ y \approx \frac{12750 \times .85}{5} \]

\[ y \approx \frac{10803.75}{5} \]

\[ y \approx 2160.75 \]

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\[ y \approx 5,657 \]
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Exercises:

1. Ryan deposited $2,100 into a savings account at his local bank. The bank offers an annual interest rate of 5%, compounded annually. What would his account balance be after 30 years if he does not deposit or withdraw any money?

\[ y = a (1 + r)^t \]
\[ y = 2100(1 + 0.05)^{30} \]
\[ y \approx \$9076 \]

2. Kelli’s mom takes a 400 mg dose of aspirin. Each hour, the amount of aspirin in a person’s system decreases by about 29%. How much aspirin is left in her system after 6 hours? Round your answer to the nearest whole number.

\[ y = a (1 - r)^t \]
\[ y = 400(1 - 0.29)^6 \]
\[ y \approx 151 \text{ mg} \]

3. A three-bedroom house in Burbville was purchased for $190,000. If housing prices are expected to increase 1.8% annually in that town:

(a) Write an explicit formula that models the price of the house in \( t \) years.

\[ y = a (1 + r)^t \]
\[ y = 190,000(1 + 0.018)^t \]
\[ a = 190,000 \]
\[ r = 1.8\% = 0.018 \]
\[ t = \text{time} \]

(b) Find the price of the house in 5 years.

\[ y = 190,000(1 + 0.018)^5 \]
\[ y \approx \$207,727 \]
\[ 207,726.78 \ldots \]
4. The country of Benin in West Africa has a population of 9.05 million people. The population is growing at a rate of 3.1% each year. Which function can be used to find the population 7 years from now?

\[ f(t) = (9.05 \times 10^6)(1 + 0.031)^t \]

- \( a = 9.05 \times 10^6 \)
- \( r = 3.1\% = 0.031 \)
- \( t = 7 \)

5. The function \( V(t) = 1350(1.017)^t \) represents the value \( V(t) \), in dollars, of a comic book \( t \) years after its purchase. The yearly rate of appreciation of the comic book is

- (1) 17%
- (2) 1.7%
- (3) 0.017%

6. The value in dollars, \( v(x) \), of a certain car after \( x \) years is represented by the equation \( v(x) = 25,000(0.86)^x \). To the nearest dollar, how much more is the car worth after 3 years than after 2 years?

\[ v(2) = 25,000(0.86)^2 \]
\[ = 18,490 \]
\[ v(3) = 25,000(0.86)^3 \]
\[ = 15,901.40 \]

1. Which table represents a function?

- [ ]

- [ ]

- [ ]

- [ ]

Function

Each input has exactly one output.

(x can't repeat)
2. The graph below represents a jogger’s speed during her 20-minute jog around her neighborhood.

Which statement best describes what the jogger was doing during the 9–12 minute interval of her jog?

(1) She was standing still.  
(2) She was increasing her speed.  
(3) She was decreasing her speed.  
(4) She was jogging at a constant rate.

3. A laboratory technician studied the population growth of a colony of bacteria. He recorded the number of bacteria every other day, as shown in the partial table below.

<table>
<thead>
<tr>
<th>t (time, in days)</th>
<th>0</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(t) (bacteria)</td>
<td>25</td>
<td>15,625</td>
<td>9,765,625</td>
</tr>
</tbody>
</table>

Which function would accurately model the technician’s data?

(1) \( f(t) = 25^t \)  
(2) \( f(t) = 25^{t+1} \)  
(3) \( f(t) = 25t \)  
(4) \( f(t) = 25(t + 1) \)

4. If \( f(x) = 3^x \) and \( g(x) = 2x + 5 \), at which value of \( x \) is \( f(x) < g(x) \)?

(1) \(-1\)  
(2) \(2\)  
(3) \(-3\)  
(4) \(4\)

(Exponential Growth)

Less than \(9.77 \times 10^6\)
Firing a piece of pottery in a kiln takes place at different temperatures for different amounts of time. The graph below shows the temperatures in a kiln while firing a piece of pottery after the kiln is preheated to 200°F.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ \frac{1640 - 1300}{5 - 2.5} = \frac{340}{2.5} = 136 \]

\[ \frac{900 - 700}{1.5 - 1} = \frac{200}{0.5} = 400 \]

\[ \frac{700 - 200}{1 - 0} = \frac{500}{1} = 500 \]

During which time interval did the temperature in the kiln show the greatest average rate of change?

(1) 0 to 1 hour
(2) 1 hour to 1.5 hours
(3) 2.5 hours to 5 hours
(4) 5 hours to 8 hours